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STABILITY OF LAMINAR FLOW OF A LIQUID AND GAS IN A HORIZONTAL CHANNEL

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It is shown that the transition from laminar to wave flow depends on the Froude number and the ratio of the equivalent thicknesses of the liquid and gas.

The interaction of a gas flow with a liquid during laminar movement in a channel is important for the design of various heat-exchanger apparatus used in power and chemical engineering. Existing regime charts and their modifications [1, 2] were obtained from visual observations and, as noted by the authors themselves, are qualitative in nature.

To more objectively classify flow regimes for two-phase flows, the study [3] proposed the use of spectral characteristics of the pressure pulsations or shear stresses on the wall. These oscillations are the result of characteristic instabilities corresponding to different modes of motion. In some cases [4], the appearance of waves and the subsequent transition to a slug regime of flow is identified with Kelvin–Helmholtz instability. Such an approach does not consider the inertial characteristics of the liquid and energy dissipation.

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Let us examine the two-dimensional laminar flow of a liquid and gas in a horizontal channel. The coordinate system is chosen so that the x axis is directed along the lower generatrix of the channel in the principal flow direction and the y axis is normal to it.

For long-wave instability, the longitudinal-velocity profile of the liquid

$$u(x,\ y,\ t) = U(x,\ t)\,(\psi(x,\ t)\,y - \varphi(x,\ t)\,y^2) \tag{1}$$

satisfies the motion equation

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} - \frac{\partial u}{\partial y}\int\limits_0^y \frac{\partial u}{\partial x}\,dy = -\frac{1}{\rho_1}\,\frac{\partial P}{\partial x} + \nu_1\frac{\partial^2 u}{\partial y^2} \tag{2}$$

with the following boundary conditions [1, 3]:

$$\begin{gathered} y = 0 \qquad u = 0, \\ y = \delta \quad \mu_1\frac{\partial u}{\partial y} = \tau = c_f\rho_2\frac{W^2}{2}, \\ P(x) = P_1(0) - \frac{1}{d}\int\limits_0^x \tau dx - \sigma\frac{\partial^2\delta}{\partial x^2} + \rho_1 g(\delta - y). \end{gathered} \tag{3}$$

In Eq. (1)

$$\varphi(x,\ t) = [2d\delta(0.5 + \delta/3d)]^{-1}, \quad \psi(x,\ t) = 2d(1 + \delta/d)\,\varphi. \tag{4}$$

The position of the liquid–gas boundary is determined by the kinematics condition [3]

$$\frac{\partial\delta}{\partial t} + \frac{\partial U\delta}{\partial x} = 0. \tag{5}$$

The stability of laminar flow of the liquid relative to small perturbations

$$\Delta\delta = \delta^*\exp i(kx - \omega t), \quad \Delta U = U^*\exp i(kx - \omega t), \tag{6}$$

which may be the result of turbulent pulsations in the gas flow, the presence of roughness, or vibrations, will be investigated on the basis of a dispersion relation.

For this, linearizing Eq. (2), averaged over the cross section, and kinematic condition (5), we obtain

$$\begin{gathered} [-i\omega + ikf_1U_0 + 2\nu_1\varphi_0]\,\Delta U + [-i\omega f_4U_0 + ik(U_0^2f_2 + g) + ik^3\sigma/\rho_1 + 2\nu_1U_0\varphi_\delta']\,\Delta\delta = 0, \\ k\delta_0\Delta U + [U_0k - \omega]\,\Delta\delta = 0, \end{gathered} \tag{7}$$

where

$$\begin{gathered} f_1 = \frac{1 + 1{,}5\eta + 0.6\eta^2}{6\,(0.5 + \eta/3)^2};\ \eta = \delta_0/d;\ f_2\delta_0 = -\frac{0.5 + 7\eta/12 + 7\eta^2/15 + 0{,}4\eta^3}{6\,(0{,}5 + \eta/3)^3}; \\ f_4\delta_0 = -\frac{0{,}5\,(0{,}5 + \eta/6 + 4\eta^2/9)}{(0.5 + \eta/3)^2};\quad U_0 = \frac{\tau\delta_0}{\mu_1}\,(0{,}5 + \eta/3). \end{gathered}$$

From the condition of nontriviality of the solution of system (7) we find the relationship between the frequency ω and the wave number k for the vibrations which occur (the dispersion relation):

$$\omega^2 - \omega kU_0(1 + f_1 - f_4\delta_0) + 2i\omega\nu_1\varphi_0 - k^4\delta_0\sigma/\rho_1 + k^2U_0^2(f_1 - f_2\delta_0 - g\delta_0/U_0^2) - 2ik\nu_1U_0(\varphi_0 - \delta_0\varphi_\delta') = 0. \tag{8}$$

The roots of dispersion relation (8) determine the interaction between small perturbations (6) and the main flow. If the real angular frequency corresponds to a wave number with an imaginary part greater than zero, the perturbations die out. When the imaginary part is negative, instability develops as the wave progresses.

Let us find the boundary between stability and instability (ω and k are real) in the form

$$\frac{\omega}{k} = U_0\frac{1 + \eta}{0{,}5 + \eta/3}, \tag{9}$$

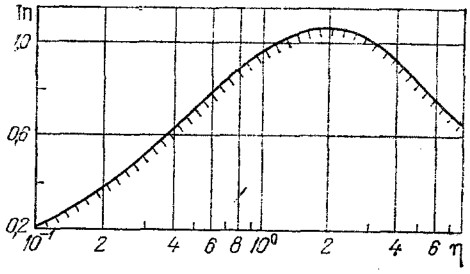

Fig. 1. Curve of neutral stability (Eq. (10)). The hatching is directed inside the region of instability.

$$\frac{k^2\delta_0\sigma}{U_0^2\rho_1}+\frac{g\delta_0}{U_0^2}=\left(\frac{1+\eta}{0,5+\eta/3}\right)^2-\left(\frac{1+\eta}{0,5+\eta/3}\right)(1+f_1-f_4\delta_0)+f_1-f_2\delta_0. \tag{10}$$

In accordance with (10), the wave characteristics of the laminar motion of the liquid and gas in a horizontal channel are characterized by the sum of two dimensionless complexes — the ratio of the surface tension over the wavelength to the inertial forces and a quantity which is the inverse of the Froude number ($\mathrm{In} = k^2\delta_0\sigma/U_0^2\rho_1 + g\delta_0/U_0^2$).

It follows from Eq. (10) that the dimensionless complex In is positive throughout the range of the parameter η (Fig. 1). If the inverse of the Froude number is less than the value obtained for In from the figure, instability develops because in this case there is a real wave number. The rate of propagation of the resulting waves is determined by Eq. (9).

It can be seen from the figure that with $\eta = 2$ (volumetric gas contant of 33%), wavelength is minimal and the frequency of oscillation of the gas—liquid boundary is maximal. This extremum is determined by the equality of the contributions of the reduction in mean velocity and increase in mass with an increase in η (drop in gas contant).

With a zero pressure gradient, the longitudinal-velocity profile (1) becomes linear, and the motion of the liquid, as in the case of Couette [5], is always stable relative to small perturbations.

Stable laminar flow can develop when the inverse of the Froude number is greater than the value of the complex In found from the figure for the corresponding η.

The theoretical results obtained here are validated by the experiments used as a basis for preparing regime charts [1-4].

NOTATION

x, y, longitudinal and transverse coordinates; u, longitudinal velocity of the liquid; U, mean velocity through the thickness of the liquid layer; t, time; ρ, density; P, pressure; ν, kinematic viscosity; δ, equivalent thickness of the liquid layer; τ, mean shear stress; c_f, friction coefficient; W, gas velocity; d, equivalent thickness (diameter) of the gas layer; g, acceleration due to gravity; σ, surface tension; Δ, symbol denoting small deviation from steady-state value; $i = \sqrt{-1}$; $k = 2\pi/\lambda$, wave number; ω, angular frequency; ν, absolute viscosity; λ, wavelength; Indices: 0, steady-state value; 1, 2, liquid and gas, respectively; the superscript * corresponds to the amplitudes of the perturbing motions.

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